
Contents

1	Introduction	1
1.1	Classification of Composites and Nanocomposites	1
1.1.1	Geometrical Classification of Composite Materials (CM) .	2
1.1.2	Classification of Mechanical Properties of CM Constituents	5
1.1.3	Classification of CM Manufacturing	8
1.2	Effective Material and Field Characteristics	8
1.3	Homogenization of Random Structure CM	12
1.4	Overview of the Book	15
2	Foundations of Solid Mechanics	17
2.1	Elements of Tensor Analysis	17
2.2	The Theory of Strains and Stresses	20
2.3	Basic Equations of Solid Mechanics	24
2.3.1	Conservation Laws, Boundary Conditions, and Constitutive Equation	24
2.3.2	The Equations of Linear Elasticity	28
2.3.3	Extremum Principles of Elastostatic	29
2.4	Basic Equations of Thermoelasticity and Electroelasticity	31
2.4.1	Thermoelasticity Equations	31
2.4.2	Electroelastic Equations	35
2.4.3	Matrix Representation of Some Symmetric Tensors	38
2.5	Symmetry of Elastic Properties	40
2.6	Basic Equations of Thermoelastoplastic Deformations	47
2.6.1	Incremental Theory of Plasticity	47
2.6.2	Deformation Theory of Plasticity	49
3	Green's Functions, Eshelby and Related Tensors	51
3.1	Static Green's Function	51
3.2	The Second Derivative of Green's Function and Related Problems	54
3.2.1	The Second Derivative of Green's Function	54
3.2.2	The Tensors Related to the Green's Function	57
3.3	Dynamic Green's and Related Functions	58
3.4	Inhomogeneity in an Elastic Medium	62

3.4.1	General Case of Inhomogeneity in an Elastic Medium	62
3.4.2	Interface Boundary Operators	65
3.5	Ellipsoidal Inhomogeneity in the Elastic Medium	67
3.6	Eshelby Tensor	71
3.6.1	Tensor Representation of Eshelby Tensor	71
3.6.2	Eshelby and Related Tensors in a Special Basis	75
3.7	Coated Ellipsoidal Inclusion	78
3.7.1	General Representation for the Concentrator Factors for Coated Heterogeneity	79
3.7.2	Single Ellipsoidal Inclusion with Thin Coating	81
3.7.3	Numerical Assessment of Thin-Layer Hypothesis	84
3.8	Related Problems for Ellipsoidal Inhomogeneity in an Infinite Medium	85
3.8.1	Conductivity Problem	85
3.8.2	Scattering of Elastic Waves by Ellipsoidal Inclusion in a Homogeneous Medium	90
3.8.3	Piezoelectric Problem	93
4	Multiscale Analysis of the Multiple Interacting Inclusions	
	Problem: Finite Number of Interacting Inclusions	95
4.1	Description of Numerical Approaches Used for Analyses of Multiple Interacting Heterogeneities	95
4.2	Basic Equations for Multiple Heterogeneities and Numerical Solution for One Inclusion	98
4.2.1	Basic Equations for Multiple Heterogeneities	98
4.2.2	The Heterogeneity v_i Inside an Imaginary Ellipsoid v_i^0 . . .	101
4.2.3	The Heterogeneity v_i Inside a Nonellipsoidal Imaginary Domain v_i^0	104
4.2.4	Estimation of Concentrator Factor Tensors by FEA	105
4.3	Volume Integral Equation Method	109
4.3.1	Regularized Representation of Integral Equations	109
4.3.2	The Iteration Method	111
4.3.3	Initial Approximation for Interacting Inclusions in an Infinite Medium	113
4.3.4	First-Order and Subsequent Approximations	117
4.3.5	Numerical Results for Two Cylindrical Inclusions in an Infinite Matrix	119
4.4	Hybrid VEE and BIE Method for Multiscale Analysis of Interacting Inclusions (Macro Problem)	120
4.4.1	Initial Approximation for the Fields Induced by a Macroinclusion	120
4.4.2	Initial Approximation in the Micro Problem	122
4.4.3	The Subsequent Approximations	123
4.4.4	Some Details of the Iteration Scheme in Multiscale Analysis	124
4.4.5	Numerical Result for a Small Inclusion Near a Large One .	126
4.4.6	Discussion	129

4.5	Complex Potentials Method for 2-D Problems	130
5	Statistical Description of Composite Materials	137
5.1	Basic Terminology and Properties of Random Variables and Random Point Fields	138
5.1.1	Random Variables	138
5.1.2	Random Point Fields	141
5.1.3	Basic Descriptors of Random Point Fields	144
5.2	Some Random Point Field Distributions	148
5.2.1	Poisson Distribution	148
5.2.2	Statistically Homogeneous Clustered Point Fields	150
5.2.3	Inhomogeneous Poisson Fields	152
5.2.4	Gibbs Point Fields	154
5.3	Ensemble Averaging of Random Structures	158
5.3.1	Ensemble Distribution Functions	159
5.3.2	Statistical Averages of Functions	164
5.3.3	Statistical Description of Indicator Functions	165
5.3.4	Geometrical Description and Averaging of Doubly and Triply Periodic Structures	171
5.3.5	Representations of ODF	173
5.4	Numerical Simulation of Random Structures	176
5.4.1	Materials and Image Analysis Procedures	178
5.4.2	Hard-Core Model	179
5.4.3	Hard-Core Shaking Model (HCSM)	180
5.4.4	Collective Rearrangement Model (CRM)	181
6	Effective Properties and Energy Methods in Thermoelasticity of Composite Materials	185
6.1	Effective Thermoelastic Properties	185
6.1.1	Hill's Condition and Representative Volume Element	185
6.1.2	Effective Elastic Moduli	188
6.1.3	Overall Thermoelastic Properties	191
6.2	Effective Energy Functions	194
6.3	Some General Exact Results	199
6.3.1	Two-Phase Composites	199
6.3.2	Polycrystals Composed of Transversally Isotropic Crystals	207
6.4	Variational Principle of Hashin and Shtrikman	209
6.5	Bounds of Effective Elastic Moduli	212
6.5.1	Hill's Bounds	212
6.5.2	Hashin-Shtrikman Bounds	217
6.5.3	Bounds of Higher Order	222
6.6	Bounds of Effective Conductivity	226
6.7	Bounds of Effective Eigenstrain	228

7	General Integral Equations of Micromechanics of Composite Materials	231
7.1	General Integral Equations for Matrix Composites of Any Structure	232
7.2	Random Structure Composites	234
7.2.1	General Integral Equation for Random Structure Composites	234
7.2.2	Some Particular Cases	237
7.2.3	Comparison with Related Equations	239
7.3	Doubly and Triply Periodical Structure Composites	241
7.4	Random Structure Composites with Long-Range Order	244
7.5	Triply Periodic Particulate Matrix Composites with Imperfect Unit Cells	246
7.6	Conclusion	248
8	Multiparticle Effective Field and Related Methods in Micromechanics of Random Structure Composites	249
8.1	Definitions of Effective Fields and Effective Field Hypotheses	250
8.1.1	Effective Fields	250
8.1.2	Approximate Effective Field Hypothesis	253
8.1.3	Closing Effective Field Hypothesis	255
8.1.4	Effective Field Hypothesis and Composites with One Sort of Inhomogeneities	255
8.2	Analytical Representation of Effective Thermoelastic Properties	258
8.2.1	Average Stresses in the Components	258
8.2.2	Effective Properties of the Composite	260
8.2.3	Some Related Multiparticle Methods	262
8.3	One-Particle Approximation of the MEFM and Mori-Tanaka Approach	264
8.3.1	One-Particle (“Quasi-Crystalline”) Approximation of MEFM	264
8.3.2	Mori-Tanaka Approach	269
8.3.3	Effective Properties Estimated via the MEF and MTM at $\mathbf{Q}_i \equiv \mathbf{Q}_i^0$	272
8.3.4	Some Methods Related to the One-Particle Approximation of the MEFM	277
8.3.5	Some Analytical Representations for Effective Moduli	280
9	Some Related Methods in Micromechanics of Random Structure Composites	283
9.1	Related Perturbation Methods	283
9.1.1	Combined MEFM–Perturbation Method	283
9.1.2	Perturbation Method for Small Concentrations of Inclusions	286
9.1.3	Perturbation Method for Weakly Inhomogeneous Media	287
9.1.4	Elastically Homogeneous Media with Random Field of Residual Microstresses	290

9.2	Effective Medium Methods	291
9.2.1	Application to Composite Materials	291
9.2.2	Analysis of Polycrystal Materials	296
9.3	Differential Methods	298
9.3.1	Scheme of the Differential Method	298
9.3.2	One-Particle Differential Method	300
9.3.3	Multiparticle Differential Method (Combination with EMM and with MEFM)	301
9.4	Estimation of Effective Properties of Composites with Nonellipsoidal Inclusions	303
9.5	Numerical Results	306
9.5.1	Composites with Spheroidal Inhomogeneities	306
9.5.2	Composites Reinforced by Nonellipsoidal Inhomogeneities with Ellipsoidal v_i^0	311
9.6	Discussion	314
10	Generalization of the MEFM in Random Structure Matrix Composites	315
10.1	Two Inclusions in an Infinite Matrix	316
10.2	Composite Material	319
10.2.1	General Representations	319
10.2.2	Some Related Integral Equations	321
10.2.3	Closing Assumption and the Effective Properties	322
10.2.4	Conditional Mean Value of Stresses in the Inclusions	323
10.3	First-order Approximation of the Closing Assumption and Effective Elastic Moduli	324
10.3.1	General Equation for the Effective Fields $\langle \bar{\sigma}_{i,j} \rangle$	324
10.3.2	Closing Assumptions for the Strain Polarization Tensor $\langle \boldsymbol{\eta}(\mathbf{y}) ; v_i, \mathbf{x}_i; v_j, \mathbf{x}_j \rangle(\mathbf{y})$	326
10.3.3	Effective Elastic Properties and Stress Concentrator Factors	329
10.3.4	Symmetric Closing Assumption	329
10.3.5	Closing Assumptions for the Effective Fields $\langle \bar{\sigma}_{i,j,k} \rangle$	330
10.4	Abandonment from the Approximative Hypothesis (10.26)	332
10.5	Some Particular Cases	334
10.5.1	Identical Aligned Inclusions	334
10.5.2	Improved Analysis of Composites with Identical Aligned Fibers	337
10.5.3	Effective Field Hypothesis	339
10.5.4	Quasi-crystalline Approximation	341
10.6	Some Particular Numerical Results	342
11	Periodic Structures and Periodic Structures with Random Imperfections	347
11.1	General Analysis of Periodic Structures and Periodic Structures with Random Imperfections	347

11.2	Triply Periodical Particular Matrix Composites in Varying External Stress Field	351
11.2.1	Basic Equation and Approximative Effective Field Hypothesis	351
11.2.2	The Fourier Transform Method	352
11.2.3	Iteration Method	353
11.2.4	Average Strains in the Components	354
11.2.5	Effective Properties of Composites	355
11.2.6	Numerical Results	356
11.3	Graded Doubly Periodical Particular Matrix Composites in Varying External Stress Field	361
11.3.1	Local Approximation of Effective Stresses	361
11.3.2	Estimation of the Nonlocal Operator via the Iteration Method	363
11.3.3	General Relations for Average Stresses and Effective Thermoelastic Properties	364
11.3.4	Some Particular Cases for Effective Properties Representations	364
11.3.5	Doubly Periodic Inclusion Field in a Finite Stringer	365
11.3.6	Numerical Results for Three-Dimensional Fields	367
11.3.7	Numerical Results for Two-Dimensional Fields	369
11.3.8	Conclusion	370
11.4	Triply Periodic Particulate Matrix Composites with Imperfect Unit Cells	371
11.4.1	Choice of the Homogeneous Comparison Medium	371
11.4.2	MEFM Accompanied by Monte Carlo Simulation	374
11.4.3	Choice of the Periodic Comparison Medium. General Scheme	376
11.4.4	The Version of MEFM Using the Periodic Comparison Medium	380
11.4.5	Concluding Remarks	383
12	Nonlocal Effects in Statistically Homogeneous and Inhomogeneous Random Structure composites	385
12.1	General Analysis of Approaches in Nonlocal Micromechanics of Random Structure Composites	385
12.2	The Nonlocal Integral Equation	390
12.3	Methods for the Solution of the Nonlocal Integral Equation	392
12.3.1	Direct Quadrature Method	392
12.3.2	The Iteration Method	392
12.3.3	The Fourier Transform Method for Statistically Homogeneous Media	393
12.4	Average Stresses in the Components and Effective Properties for Statistically Homogeneous Media	396
12.4.1	Differential Representations	396
12.4.2	The Reduction of Integral Overall Constitutive Equations to Differential Ones	397

12.4.3	“Quasi-crystalline” Approximation	398
12.4.4	Numerical Analysis of Nonlocal Effects for Statistically Homogeneous Composites	399
12.5	Effective Properties of Statistically Inhomogeneous Media	403
12.5.1	Local Effective Properties of FGMs	403
12.5.2	Elastically Homogeneous Composites	406
12.5.3	Numerical Results of Estimation of Effective Properties of FGMs	407
12.5.4	Perturbation Method	412
12.5.5	Combined MEFM-Perturbation Method	413
12.5.6	The MEF Method	413
12.6	Concluding Remarks	414
13	Stress Fluctuations in Random Structure Composites	417
13.1	Perturbation Method	419
13.1.1	Exact Representation for First and Second Moments of Stresses Averaged over the Phase Volumes	419
13.1.2	Local Fluctuation of Stresses	423
13.1.3	Correlation Function of Stresses	423
13.1.4	Numerical Results and Discussions	425
13.2	Method of Integral Equations	427
13.2.1	Estimation of the Second Moment of Effective Stresses	427
13.2.2	Implicit Representations for the Second Moment of Stresses	428
13.2.3	Explicit Estimation of Second Moments of Stresses Inside the Phases	430
13.2.4	Numerical Estimation of the Second Moments of Stresses in the Phases	431
13.2.5	Related Method of Estimations of the Second Moments of Stresses	433
13.3	Elastically Homogeneous Composites with Randomly Distributed Residual Microstresses	434
13.3.1	The Conditional Average of the Stresses Inside the Components	435
13.3.2	The Second Moment Stresses Inside the Phases	435
13.3.3	Numerical Evaluation of Statistical Residual Stress Distribution in Elastically Homogeneous Media	438
13.4	Stress Fluctuations Near a Crack Tip in Elastically Homogeneous Materials with Randomly Distributed Residual Microstresses	440
13.4.1	The Average and Conditional Mean Values of SIF for Isolated Crack in a Composite Material	441
13.4.2	Conditional Dispersion of SIF for a Crack in the Composite Medium	444
13.4.3	Crack in a Finite Inclusion Cloud	445
13.4.4	Numerical Estimation of the First and Second Statistical Moments of Stress Intensity Factors	447

13.5	Concluding Remarks	449
14	Random Structure Matrix Composites in a Half-Space	451
14.1	General Analysis of Approaches in Micromechanics of Random Structure Composites in a Half-space	451
14.2	General Integral Equation, Definitions of the Nonlocal Effective Properties, and Averaging Operations	455
14.3	Finite Number of Inclusions in a Half-Space	458
14.3.1	A Single Inclusion Subjected to Inhomogeneous Effective Stress	458
14.3.2	Two Inclusions	459
14.4	Nonlocal Effective Operators of Thermoelastic Properties of Microinhomogeneous Half-Space	462
14.4.1	Dilute Concentration of Inclusions	462
14.4.2	c^2 Order Accurate Estimation of Effective Thermoelastic Properties	463
14.4.3	Quasi-crystalline Approximation	465
14.4.4	Influence of a Correlation Hole v_{ij}^0	468
14.5	Statistical Properties of Local Residual Microstresses in Elastically Homogeneous Half-Space	469
14.5.1	First Moment of Stresses in the Inclusions	469
14.5.2	Limiting Case for a Statistically Homogeneous Medium	471
14.5.3	Stress Fluctuations Inside the Inclusions	472
14.6	Numerical Results	474
15	Effective Limiting Surfaces in the Theory of Nonlinear Composites	481
15.1	Local Limiting Surface	482
15.1.1	Local Limiting Surface for Bulk Stresses	482
15.1.2	Local Limiting Surface for Interface Stresses	483
15.1.3	Fracture Criterion for an Isolated Crack	485
15.2	Effective Limiting Surface	485
15.2.1	Utilizing Fluctuations of Bulk Stresses Inside the Phases	485
15.2.2	Utilizing Interface Stress Fluctuations	487
15.2.3	Effective Fracture Surface for an Isolated Crack in the Elastically Homogeneous Medium with Random Residual Microstresses	489
15.2.4	Scheme of Simple Probability Model of Composite Fracture	490
15.3	Numerical Results	492
15.3.1	Utilizing Fluctuations of Bulk Stresses Inside the Phases	492
15.3.2	Utilizing Interface Stress Fluctuations	499
15.3.3	Effective Energy Release Rate and Fracture Probability	501
15.4	Concluding Remarks	503

16 Nonlinear Composites	505
16.1 Nonlinear Elastic Composites	506
16.1.1 Popular Linearization Scheme	506
16.1.2 Modified Linearization Scheme	510
16.2 Deformation Plasticity Theory of Composite Materials	513
16.2.1 General Scheme	513
16.2.2 Elastoplastic Deformation of Composites with an Incompressible Matrix	516
16.2.3 General Case of Elastoplastic Deformation	517
16.3 Power-Law Creep	517
16.4 Elastic–Plastic Behavior of Elastically Homogeneous Materials with Random Residual Microstresses	521
16.4.1 Leading Equations and Elastoplastic Deformations	521
16.4.2 Numerical Results for Temperature-Independent Properties	525
16.5 A Local Theory of Elastoplastic Deformations of Metal Matrix Composites	527
16.5.1 Geometrical Structure of the Components	527
16.5.2 Average Stresses Inside the Components and Overall Elastic Moduli	529
16.5.3 Elastoplastic Deformation	530
16.5.4 Numerical Results	532
17 Some related problems	537
17.1 Conductivity	537
17.1.1 Basic Equations and General Analysis	537
17.1.2 Perturbation Methods	539
17.1.3 Self-Consistent Methods	543
17.1.4 Nonlinear and Nonlocal Properties	548
17.2 Thermoelastoelectricity of Composites	549
17.2.1 General Analysis	549
17.2.2 Generalized Hill’s Conditions and Effective Properties	553
17.2.3 Effective Energy Functions	555
17.2.4 Two-Phase Composites	556
17.2.5 Discontinuities of Generalized Fields at the Interface Between Components	557
17.2.6 Phase-Averaged First and Second Moments of the Field Σ	558
17.3 Wave Propagation in a Composite Material	561
17.3.1 General Integral Equations and Effective Fields	561
17.3.2 Fourier Transform of Effective Wave Operator	564
17.3.3 Effective Wave Operator for Composites with Spherical Isotropic Inclusions	568

18 Multiscale Mechanics of Nanocomposites	571
18.1 Elements of Molecular Dynamic (MD) Simulation	571
18.1.1 General Analysis of MD Simulation of Nanocomposites ...	571
18.1.2 Foundations of MD Simulation and Their Use in Estimation of Elastic Moduli	574
18.1.3 Interface Modeling of NC	576
18.2 Bridging Nanomechanics to Micromechanics in Nanocomposites .	578
18.2.1 General Representations for the Local Effective Moduli ...	579
18.2.2 Generalization of Popular Micromechanical Methods to the Estimations of Effective Moduli of NCs	580
18.3 Modeling of Nanofiber NCs in the Framework of Continuum Mechanics	582
18.3.1 Statistical Description of NCs with Prescribed Random Orientation of NTs	582
18.3.2 One Nanofiber Inside an Infinite Matrix	583
18.3.3 Numerical Results for NCs Reinforced with Nanofibers ...	586
18.4 Modeling of Clay NCs in the Framework of Continuum Mechanics	590
18.4.1 Existing Modeling of Clustered Materials and Clay NCs .	590
18.4.2 Estimations of Effective Thermoelastic Properties and Stress Concentrator Factors of Clay NCs via the MEF ...	593
18.4.3 Numerical Solution for a Single Cluster in an Infinite Medium	596
18.4.4 Numerical Estimations of Effective Properties of Clay NCs	598
18.5 Some Related Problems in Modeling of NCs Reinforced with NFs and Nanoplates	602
19 Conclusion. Critical Analysis of Some Basic Concepts of Micromechanics	607
A Appendix	611
A.1 Parametric Representation of Rotation Matrix	611
A.2 Second and Fourth-Order Tensors of Special Structures	613
A.2.1 E-basis	613
A.2.2 P-basis	614
A.2.3 B-basis	617
A.3 Analytical Representation of Some Tensors	619
A.3.1 Exterior-Point Eshelby Tensor	619
A.3.2 Some Tensors Describing Fluctuations of Residual Stresses	621
A.3.3 Integral Representations for Stress Intensity Factors	622
References	623