

# Contents

<b>1</b>	<b>Basic Concepts and the Discovery of Solitons</b> .....	1
1.1	A look at linear and nonlinear signatures.....	1
1.2	Discovery of the solitary wave .....	3
1.3	Discovery of the soliton .....	7
1.4	The soliton concept in physics .....	11
<b>2</b>	<b>Linear Waves in Electrical Transmission Lines</b> .....	12
2.1	Linear nondispersive waves .....	12
2.2	Sinusoidal-wave characteristics .....	15
2.2.1	Wave energy density and power.....	18
2.3	The group-velocity concept.....	19
2.4	Linear dispersive waves.....	21
2.4.1	Dispersive transmission lines .....	21
2.4.2	Electrical network .....	23
2.4.3	The weakly dispersive limit.....	26
2.5	Evolution of a wavepacket envelope.....	27
2.6	Dispersion-induced wavepacket broadening.....	31
Appendix 2A.	General solution for the envelope evolution.....	34
Appendix 2B.	Evolution of the envelope of a Gaussian wavepacket...	35
<b>3</b>	<b>Solitons in Nonlinear Transmission Lines</b> .....	37
3.1	Nonlinear and dispersionless transmission lines.....	37
3.2	Combined effects of dispersion and nonlinearity.....	41
3.3	Electrical solitary waves and pulse solitons.....	42
3.4	Laboratory experiments on pulse solitons.....	46
3.4.1	Experimental arrangement.....	46
3.4.2	Series of experiments .....	48
3.5	Experiments with a pocket version of the electrical network.....	52

3.6	Nonlinear transmission lines in the microwave range .....	56
Appendix 3A.	Calculation of the effect of nonlinearity on wave propagation .....	58
Appendix 3B.	Derivation of the solitary-wave solution .....	60
Appendix 3C.	Derivation of the KdV equation and its soliton solution	62
Appendix 3D.	Details of the electronics: switch driver and pulse generator .....	64
<b>4</b>	<b>More on Transmission-Line Solitons.....</b>	<b>65</b>
4.1	Lattice solitons in the electrical Toda network .....	65
4.1.1	Lattice solitons.....	67
4.2	Experiments on lattice solitons .....	68
4.2.1	Collisions of two lattice solitons moving in opposite directions.....	70
4.2.2	The Fermi-Pasta-Ulam recurrence phenomenon.....	70
4.3	Periodic wavetrains in transmission lines .....	71
4.3.1	The solitary wave limit and sinusoidal limit of the cnoidal wave.....	72
4.4	Modulated waves and the nonlinear dispersion relation .....	72
4.5	Envelope and hole solitons .....	74
4.5.1	Experiments on envelope and hole solitons .....	76
4.6	Modulational instability.....	77
4.7	Laboratory experiments on modulational instability .....	82
4.7.1	Model equations .....	82
4.7.2	Experiments.....	84
4.8	Modulational instability of two coupled waves.....	86
4.9	Microwave solitons in magnetic transmission lines.....	88
4.9.1	Nonlinear spin waves.....	88
4.9.2	NLS model equation for spin waves.....	88
4.9.3	Observation of magnetic envelope solitons.....	89
4.10	Solitons and signal processing.....	91
Appendix 4A.	Periodic wavetrain solutions .....	93
Appendix 4B.	The Jacobi elliptic functions.....	95
4B.1	Asymptotic limits.....	96
4B.2	Derivatives and integrals.....	98
Appendix 4C.	Envelope and hole soliton solutions.....	98

<b>5</b>	<b>Hydrodynamic Solitons</b> .....	103
5.1	Equations for surface water waves.....	103
5.1.1	Reduced fluid equations .....	104
5.2	Small-amplitude surface gravity waves.....	100
5.3	Linear shallow- and deep-water waves.....	108
5.3.1	Shallow-water waves .....	108
5.3.2	Deep-water waves .....	109
5.4	Surface-tension effects: capillary waves .....	110
5.5	Solitons in shallow water .....	112
5.6	Experiments on solitons in shallow water .....	115
5.6.1	Experimental arrangement.....	116
5.6.2	Experiments.....	116
5.7	Stokes waves and soliton wavepackets in deep water .....	120
5.7.1	Stokes waves .....	120
5.7.2	Soliton wavepackets .....	121
5.7.3	Experiments on solitons in deep water .....	122
5.8	Experiments on modulational instability in deep water .....	123
5.9	Some applications of the KdV model.....	126
5.9.1	Blood pressure wave propagation.....	126
5.9.2	Nonlinear modes of liquid drops.....	127
Appendix 5A.	Basic equations of fluid mechanics.....	127
5A.1	Conservation of mass.....	127
5A.2	Conservation of momentum.....	129
5A.3	Conservation of entropy.....	130
Appendix 5B.	Basic definitions and approximations.....	130
5B.1	Streamline .....	130
5B.2	Irrotational and incompressible flow .....	131
5B.3	Two-dimensional flow: the stream function.....	132
5B.4	Boundary conditions.....	134
5B.5	Surface tension .....	135
Appendix 5C.	Derivation of the KdV equation: the perturbative approach.....	136
Appendix 5D.	Derivation of the nonlinear dispersion relation.....	139
Appendix 5E.	Details of the probes and the electronics.....	142

<b>6</b>	<b>Mechanical Solitons</b> .....	143
6.1	An experimental mechanical transmission line .....	143
6.1.1	General description of the line .....	143
6.1.2	Construction of the line.....	145
6.2	Mechanical kink solitons .....	145
6.2.1	Linear waves in the low-amplitude limit.....	146
6.2.2	Large amplitude waves: kink solitons.....	147
6.2.3	Lorentz contraction of the kink solitons .....	149
6.3	Particle properties of the kink solitons.....	151
6.4	Kink–kink and kink–antikink collisions.....	152
6.5	Breather solitons .....	154
6.6	Experiments on kinks and breathers .....	156
6.7	Helical waves, or kink array.....	157
6.8	Dissipative effects.....	159
6.9	Envelope solitons .....	161
6.10	Lattice effects.....	163
6.10.1	Pocket version of the pendulum chain, lattice effects....	163
6.10.2	Pendulum chain with weak coupling.....	164
6.11	A mechanical transmission line with two equilibrium states.....	165
6.11.1	Periodic and double-well substrate potentials.....	165
6.11.2	General description of the mechanical chain.....	166
6.11.3	Kink-soliton solutions.....	169
6.11.4	Compacton-like kinks or compactons.....	170
6.11.5	Experiments.....	173
6.12	Solitons, compactons and nanopterons.....	175
Appendix 6A.	Kink-soliton and antikink-soliton solutions.....	178
Appendix 6B.	Calculation of the energy and the mass of a kink soliton .....	179
Appendix 6C.	Solutions for kink–kink and kink–antikink collisions, and breathers.....	180
6C.1	Kink solutions .....	182
6C.2	Kink–kink collisions.....	182
6C.3	Breather solitons.....	183
6C.4	Kink–antikink collision.....	184
Appendix 6D.	Solutions for helical waves.....	185
Appendix 6E.	Pendulum with torsion and gravity.....	187

Appendix 6F	Model equation for the pendulum chain.....	187
<b>7</b>	<b>Fluxons in Josephson Transmission Lines</b> .....	189
7.1	The Josephson effect in a short junction .....	189
7.1.1	The small Josephson junction .....	190
7.2	The long Josephson junction as a transmission line .....	192
7.3	Dissipative effects .....	196
7.4	Experimental observations of fluxons .....	198
7.4.1	Indirect observation .....	198
7.4.2	Direct observation .....	199
7.4.3	Lattice effects .....	201
Appendix 7A.	Josephson equations .....	201
<b>8</b>	<b>Solitons in Optical Fibers</b> .....	203
8.1	Optical-fiber characteristics .....	203
8.1.1	Linear dispersive effects.....	204
8.1.2	Nonlinear effects .....	206
8.1.3	Effect of losses .....	207
8.2	Wave-envelope propagation .....	208
8.3	Bright and dark solitons .....	210
8.3.1	Bright solitons .....	211
8.3.2	Dark solitons.....	213
8.4	Experiments on optical solitons .....	214
8.5	Perturbations and soliton communications.....	216
8.5.1	Effect of losses .....	216
8.5.2	Soliton communications .....	217
8.6	Modulational instability of coupled waves .....	218
8.7	A look at quantum-optical solitons.....	219
8.8	Some other kinds of optical solitons: spatial solitons.....	221
Appendix 8A.	Electromagnetic equations in a nonlinear medium.....	222
<b>9</b>	<b>The Soliton Concept in Lattice Dynamics</b> .....	225
9.1	The one-dimensional lattice in the continuum approximation.....	225
9.2	The quasi-continuum approximation for the monatomic lattice....	230
9.3	The Toda lattice .....	232
9.4	Envelope solitons and localized modes.....	233

9.5	The one-dimensional lattice with transverse nonlinear modes . . . .	235
9.6	Motion of dislocations in a one-dimensional crystal . . . . .	238
9.7	The one-dimensional lattice model for structural phase transitions. . . . .	239
9.7.1	The order–disorder transition . . . . .	241
9.7.2	The displacive transition. . . . .	242
9.8	Kink-soliton solutions for generalized on-site potentials. . . . .	244
9.9	A lattice model with an exact kink-soliton solution. . . . .	247
9.10	Energy localization in nonlinear lattices. . . . .	250
9.10.1	Self-trapped states: polaron and conformon. . . . .	250
9.10.2	Intrinsic localized modes or discrete breathers. . . . .	251
9.11	Observation of discrete breathers. . . . .	253
9.11.1	Discrete pendulum chains. . . . .	253
9.11.2	Mechanical chain with torsion and gravity. . . . .	254
9.11.3	A chain of magnetic pendulums. . . . .	256
Appendix 9A.	Solutions for transverse displacements . . . . .	257
Appendix 9B.	Kink-soliton or domain-wall solutions . . . . .	259
Appendix 9C	Construction of a double-well potential. . . . .	260
<b>10</b>	<b>A Look at Some Remarkable Mathematical Techniques . . . . .</b>	<b>262</b>
10.1	Lax equations and the inverse scattering transform method. . . . .	262
10.1.1	The Fourier-transform method for linear equations . . . . .	263
10.1.2	The Lax pair for nonlinear evolution equations. . . . .	264
10.2	The KdV equation and the spectral problem . . . . .	266
10.3	Time evolution of the scattering data. . . . .	267
10.3.1	Discrete eigenvalues. . . . .	267
10.3.2	Continuous spectrum . . . . .	269
10.4	The inverse scattering problem . . . . .	270
10.4.1	Discrete spectrum only: soliton solution. . . . .	271
10.5	Response of the KdV model to an initial disturbance . . . . .	273
10.5.1	The delta function potential . . . . .	273
10.5.2	The rectangular potential well. . . . .	274
10.5.3	The sech-squared potential well . . . . .	274
10.6	The inverse scattering transform for the NLS equation. . . . .	275
10.7	The Hirota method for the KdV equation . . . . .	277
10.8	The Hirota method for the NLS equation . . . . .	280

<b>11 Diffusive solitons.....</b>	<b>284</b>
11.1 Combined effects of dissipation and nonlinearity .....	285
11.1.1 A diffusive electrical transmission line.....	285
11.1.2 Linear diffusive waves.....	287
11.1.3 Kink-shaped diffusive solitons.....	288
11.1.4 Experiments on electrical diffusive solitons.....	290
11.2 Reaction diffusion processes .....	291
11.2.1 Reaction diffusion equations.....	291
11.2.2 A chemical model with reaction diffusion.....	293
11.2.3 An electrical lattice with reaction diffusion.....	296
11.2.4 Experiments with an electrical lattice.....	298
11.3 A mechanical analog with diffusive solitons.....	299
11.3.1 Chain with flexion and gravity.....	299
11.3.2 Experimental chain .....	300
11.4 Reaction diffusion processes in lattices.....	301
11.4.1 Propagation failure .....	301
11.4.2 Discrete reaction diffusion model with exact solution...	302
Appendix 11A. Derivation of the Burgers equation. ....	303
Appendix 11B. Solution of the reaction diffusion equation.....	304
Appendix 11C. Equation of motion of an Euler strut.....	305
 <b>References .....</b>	 <b>307</b>
 <b>Subject Index .....</b>	 <b>325</b>

