
Contents

1 Jorgensen's picture of quasifuchsian punctured torus groups	1
1.1 Punctured torus groups, Ford domains and EPH-decompositions	2
1.2 Jorgensen's theorem for quasifuchsian punctured torus groups (I)	7
1.3 Jorgensen's theorem for quasifuchsian punctured torus groups (II)	12
1.4 The topological ideal polyhedral complex $\text{Trg}(\nu)$ dual to $\text{Spine}(\nu)$	13
2 Fricke surfaces and $PSL(2, \mathbb{C})$-representations	15
2.1 Fricke surfaces and their fundamental groups	16
2.2 Type-preserving representations	21
2.3 Markoff maps and type-preserving representations	26
2.4 Markoff maps and complex probability maps	29
2.5 Miscellaneous properties of discrete groups	33
3 Labeled representations and associated complexes	37
3.1 The complex $\mathcal{L}(\rho, \sigma)$ and upward Markoff maps	38
3.2 The complexes $\mathcal{L}(\rho, \Sigma)$ and $\mathcal{L}(\Sigma)$	41
3.3 Labeled representation $\rho = (\rho, \nu)$ and the complexes $\mathcal{L}(\rho)$ and $\mathcal{L}(\nu)$	44
3.4 Virtual Ford domain	44
4 Chain rule and side parameter	49
4.1 Chain rule for isometric circles	50
4.2 Side parameter	56
4.3 ϵ -terminal triangles	66
4.4 Basic properties of ϵ -terminal triangles	70
4.5 Relation between side parameters at adjacent triangles	78
4.6 Transition of terminal triangles	82

XLII Contents

4.7	Proof of Lemma 4.5.5	89
4.8	Representations which are weakly simple at σ	95
5	Special examples	101
5.1	Real representations	102
5.2	Isosceles representations and thin labels	106
5.3	Groups generated by two parabolic transformations	117
5.4	Imaginary representations	126
5.5	Representations with accidental parabolic/elliptic transformations	127
6	Reformulation of Main Theorem 1.3.5 and outline of the proof	133
6.1	Reformulation of Main Theorem 1.3.5	134
6.2	Route map of the proof of Modified Main Theorem 6.1.11	136
6.3	The cellular structure of $\partial Eh(\rho)$	138
6.4	Applying Poincare's theorem on fundamental polyhedra	142
6.5	Proof of Theorem 6.1.8 (Good implies quasifuchsian)	144
6.6	Structure of the complex Δ_E and the proof of Theorem 6.1.12	147
6.7	Characterization of $\Sigma(\nu)$ for good labeled representations	151
7	Openness	155
7.1	Hidden isometric hemispheres	155
7.2	Proof of Proposition 6.2.1 (Openness) - Thick Case -	159
7.3	Proof of Proposition 6.2.1 (Openness) - Thin case -	165
8	Closedness	171
8.1	Proof of Proposition 6.2.3 (SameStratum)	172
8.2	Proof of Proposition 6.2.7 (Convergence)	178
8.3	Route map of the proof of Proposition 6.2.4 (Closedness)	180
8.4	Reduction of Proposition 8.3.5 - The condition HausdorffConvergence -	182
8.5	Classification of simplices of $\mathcal{L}(\nu)$	184
8.6	Proof of Proposition 8.4.4 ($\overline{F}_\infty(\xi) \subset \partial \overline{Eh}(\rho_\infty, \mathcal{L}_0)$)	185
8.7	Accidental parabolic transformation	187
8.8	Proof of Proposition 8.4.5 - length 1 case -	189
8.9	Proof of Proposition 8.4.5 - length ≥ 2 case - (Step 1)	191
8.10	Proof of Proposition 8.4.5 - length ≥ 2 case - (Step 2)	203
8.11	Proof of Proposition 8.4.5 - length ≥ 2 case - (Step 3)	206
8.12	Proof of Proposition 8.3.6	209
9	Algebraic roots and geometric roots	215
9.1	Algebraic roots	215
9.2	Unique existence of the geometric root	227
9.3	Continuity of roots and continuity of intersections	229

Contents XLIII

A Appendix	233
A.1 Basic facts concerning the Ford domain	233
References	239
Notation	245
Index	249

