

Contents

Preface

vii

Part 1: Rough Sets

| | | |
|----------|--|----------|
| 1 | Rough Set Theory: An Introduction | 3 |
| 1.1 | Knowledge representation | 3 |
| 1.2 | Information systems | 5 |
| 1.3 | Exact sets, rough sets, approximations | 6 |
| 1.4 | Set–algebraic structures | 9 |
| 1.5 | Topological structures | 9 |
| 1.6 | Logical aspects of rough sets | 11 |
| 1.6.1 | Dependencies | 11 |
| 1.6.2 | Modal aspects of rough sets | 15 |
| 1.6.3 | Many–valued logics for rough sets | 16 |
| 1.7 | Decision systems | 16 |
| 1.8 | Knowledge reduction | 18 |
| 1.8.1 | Reducts via boolean reasoning: discernibility approach | 19 |
| 1.8.2 | Reducts in decision systems | 20 |
| 1.8.3 | Rough membership functions | 21 |
| 1.9 | Similarity based techniques | 22 |
| 1.9.1 | General approximation spaces | 23 |
| 1.9.2 | Rough mereology | 24 |
| 1.9.3 | Rough mereology in complex information systems: in search of features in distributed systems | 26 |
| 1.10 | Generalized and approximate reducts | 31 |
| 1.10.1 | Frequency based reducts | 31 |
| 1.10.2 | Local reducts | 34 |
| 1.10.3 | Dynamic reducts | 35 |
| 1.10.4 | Generalized dynamic reducts | 35 |
| 1.10.5 | Genetic and hybrid algorithms in reduct computation | 36 |
| 1.11 | Template techniques | 38 |
| 1.11.1 | Templates | 39 |

| | | |
|----------------------------------|---|------------|
| 1.11.2 | Template goodness measures | 40 |
| 1.11.3 | Searching for optimal descriptors | 42 |
| 1.12 | Discretization | 42 |
| 1.12.1 | Value partition via cuts | 43 |
| 1.12.2 | Heuristics | 44 |
| 1.13 | Selected Bibliography on Rough Sets | 50 |
| Part 2: Prerequisites | | |
| 2 | The Sentential Calculus | 95 |
| 2.1 | Introduction | 95 |
| 2.2 | Functors | 96 |
| 2.3 | Meaningful expressions | 97 |
| 2.4 | The Sentential Calculus | 98 |
| 2.5 | Exemplary derivations in (L) | 101 |
| 2.6 | Completeness of (L) | 104 |
| 2.7 | The sequent approach | 107 |
| 2.7.1 | Axioms and inference rules for the sequent calculus of the sentential calculus | 107 |
| 3 | Logical Theory of Approximations | 121 |
| 3.1 | Introduction | 121 |
| 3.2 | Figures of syllogisms | 121 |
| 3.3 | Syllogistic as a deductive system | 125 |
| 3.4 | Selected syllogisms | 129 |
| 4 | Many-valued Sentential Calculi | 137 |
| 4.1 | Introduction | 137 |
| 4.2 | A formal development | 138 |
| 4.3 | Consistent sets of meaningful expressions | 140 |
| 4.4 | Completeness | 141 |
| 4.5 | n -valued logics | 145 |
| 4.6 | 4-valued logic: Modalities | 148 |
| 4.7 | Modalities | 151 |
| 5 | Propositional Modal Logic | 159 |
| 5.1 | Introduction | 159 |
| 5.2 | The system K | 160 |
| 5.3 | The system T | 166 |
| 5.4 | The system S4 | 166 |
| 5.5 | The system S5 | 167 |

| | | |
|----------|--|------------|
| 6 | Set Theory | 173 |
| 6.1 | Introduction | 173 |
| 6.2 | Naive set theory | 174 |
| 6.2.1 | Algebra of sets | 175 |
| 6.3 | A formal approach | 180 |
| 6.4 | Relations and functions | 183 |
| 6.4.1 | Algebra of relations | 184 |
| 6.5 | Orderings | 186 |
| 6.6 | Lattices and Boolean algebras | 189 |
| 6.7 | Infinite sets | 190 |
| 6.8 | Well-ordered sets | 192 |
| 6.9 | Finite versus infinite sets | 194 |
| 6.10 | Equipotency | 196 |
| 6.11 | Countable Sets | 198 |
| 6.12 | Filters and ideals | 200 |
| 6.13 | Equivalence, tolerance | 201 |
| 6.13.1 | Tolerance relations | 204 |
| 7 | Topological Structures | 213 |
| 7.1 | Introduction | 213 |
| 7.2 | Metric spaces | 213 |
| 7.3 | Topological Cartesian products | 218 |
| 7.4 | Compactness in metric spaces | 220 |
| 7.5 | Completeness in metric spaces | 221 |
| 7.6 | General topological spaces | 225 |
| 7.7 | Regular sets | 227 |
| 7.8 | Compactness in general spaces | 229 |
| 7.9 | Continuity | 233 |
| 7.10 | Topologies on subsets | 235 |
| 7.11 | Quotient spaces | 236 |
| 7.12 | Hyperspaces | 236 |
| 7.12.1 | Topologies on closed sets | 237 |
| 7.13 | Čech topologies | 242 |
| 8 | Algebraic Structures | 251 |
| 8.1 | Introduction | 251 |
| 8.2 | Lattices | 251 |
| 8.3 | Distributive lattices | 254 |
| 8.4 | Pseudo-complement | 256 |
| 8.5 | Stone lattices | 258 |
| 8.6 | Complement | 258 |
| 8.7 | Boolean algebras | 260 |
| 8.8 | Filters on lattices | 261 |
| 8.9 | Filters on Boolean algebras | 264 |
| 8.10 | Pseudo-Boolean algebras | 266 |

| | | |
|--|--|------------|
| 9 | Predicate Calculus | 273 |
| 9.1 | Introduction | 273 |
| 9.2 | A formal predicate calculus | 274 |
| 9.3 | The Lindenbaum-Tarski algebra | 278 |
| 9.4 | Completeness | 282 |
| 9.4.1 | Calculus of open expressions | 286 |
| 9.5 | Calculus of unary predicates | 286 |
| 9.6 | Fractional truth values | 287 |
| 9.7 | Intuitionistic propositional logic | 289 |
| 9.7.1 | Gentzen-type formalization of predicate and intuitionistic calculi | 291 |
| Part 3: Mathematics of Rough Sets | | |
| 10 | Independence, Approximation | 299 |
| 10.1 | Introduction | 299 |
| 10.2 | Independence | 300 |
| 10.2.1 | Functional dependence | 302 |
| 10.2.2 | An abstract view on independence | 306 |
| 10.2.3 | Dependence spaces | 308 |
| 10.2.4 | Independence | 309 |
| 10.2.5 | Dependence | 312 |
| 10.2.6 | Interpretation in information systems | 314 |
| 10.3 | Classification/approximation spaces | 315 |
| 10.3.1 | Approximation spaces of an information system | 319 |
| 10.4 | Partial dependence | 321 |
| 11 | Topology of Rough Sets | 331 |
| 11.1 | Introduction | 331 |
| 11.2 | Π_0 -rough sets | 332 |
| 11.3 | Metrics on rough sets | 334 |
| 11.3.1 | Some examples | 339 |
| 11.4 | Almost rough sets | 341 |
| 11.5 | Fractals, Approximate Collage | 345 |
| 11.5.1 | Fractals | 345 |
| 11.5.2 | The Approximate Collage Theorem | 353 |
| 12 | Algebra and Logic of Rough Sets | 361 |
| 12.1 | Introduction | 361 |
| 12.2 | Algebraic structures via rough sets | 362 |
| 12.2.1 | Nelson algebras of rough sets | 362 |
| 12.2.2 | Heyting algebras of rough sets | 365 |
| 12.2.3 | Stone algebras of rough sets | 367 |
| 12.3 | Łukasiewicz algebras of rough sets | 372 |
| 12.3.1 | Wajsberg algebras | 375 |

| | | |
|--------|---|-----|
| 12.3.2 | Post algebra representation of rough sets | 387 |
| 12.4 | A logic of indiscernibility | 389 |
| 12.4.1 | The syntax of FD-logic | 389 |
| 12.4.2 | Semantics of FD-logic | 391 |
| 12.5 | Information logics | 394 |
| 12.5.1 | The logic IL | 396 |
| 12.5.2 | A canonical model | 399 |

Part 4: Rough vs. Fuzzy

| | | |
|-----------|--|------------|
| 13 | Infinite-valued Logical Calculi | 413 |
| 13.1 | Introduction | 413 |
| 13.2 | Syntax of L_∞ | 415 |
| 13.3 | Semantics of L_∞ | 421 |
| 13.3.1 | Polynomials, polynomial formulae | 422 |
| 13.4 | Fuzzy logics of sentences | 427 |
| 13.4.1 | Basic ingredients of a fuzzy logic | 429 |
| 13.4.2 | The Lukasiewicz residuated lattice | 431 |
| 13.4.3 | Filters on residuated lattices | 434 |
| 13.4.4 | Syntax and semantics of a fuzzy sentential calculus | 436 |
| 13.4.5 | Syntax and semantics at work | 439 |
| 13.4.6 | Completeness of the fuzzy sentential calculus | 441 |
| 13.4.7 | Discrete Lukasiewicz residuated lattices | 449 |
| 13.4.8 | 3 - Lukasiewicz algebras vs. Lindenbaum-Tarski algebras of the fuzzy sentential calculus | 450 |
| 14 | From Rough to Fuzzy | 465 |
| 14.1 | Introduction | 465 |
| 14.2 | Triangular norms | 466 |
| 14.3 | Rough fuzzy and fuzzy rough sets | 479 |
| 14.3.1 | Rough fuzzy sets | 481 |
| 14.3.2 | Fuzzy rough sets | 483 |
| 14.4 | Brouwer-Zadeh lattices | 491 |
| | Bibliography | 501 |
| | Index | 521 |
| | List of Symbols | 531 |